

Part-I

10×4=40

1. The solution of the diameter of the ellipse $2x^2 + 3y^2 = 6$, which bisects all chords parallel to $3x + 4y = 5$ is

(a) $9y - 8x = 0$

(b) $8y - 9x = 0$

(c) $4x + 3y = 6$

(d) $8y + 9x = 0$.

2. The value of $(3x + 4i)^{-1/2} + (3x - 4i)^{-1/2}$ is

(a) $\frac{4}{3}$

(b) $\frac{5}{4}$

(c) $\frac{3}{4}$

(d) $\frac{4}{5}$.

3. A function $f(x)$ is defined by the equations

$$\begin{aligned} f(x) &= 0 && \text{for } x^2 > 1 \\ &= 1 && \text{for } x^2 < 1 \\ &= \frac{1}{2} && \text{for } x^2 = 1. \end{aligned}$$

The function $f(x)$ is

(a) continuous at $x = \pm 1$

(b) discontinuous at $x = \pm 1$

(c) differentiable at $x = \pm 1$

(d) differentiable at $x = \pm 2$.

4. $\lim_{x \rightarrow \infty} \frac{15x^7 + 12x + 17}{5x^7 + 9x^2 + 12}$ is equal to

- (a) 0
- (b) 2
- (c) 3
- (d) 4.

5. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, the value of $\frac{dy}{dx}$ is

- (a) $\sqrt{\frac{1-y^2}{1-x^2}}$
- (b) $\sqrt{\frac{1-x^2}{1-y^2}}$
- (c) $\sqrt{\frac{1-y}{1-x}}$
- (d) $\sqrt{\frac{1-x}{1-y}}$.

6. The function $f(x) = \sin x (1 + \cos x)$ has

- (a) a minimum at $x = \pi$
- (b) a minimum at $x = \frac{1}{3}\pi$
- (c) neither a maximum nor a minimum at $x = \frac{1}{3}\pi$
- (d) a maximum at $x = \frac{1}{3}\pi$.

7. $\int \frac{\sin x}{\sqrt{1+\sin x}} dx$ is equal to

- (a) $\sqrt{2} \log \tan \left(\frac{\pi}{8} + \frac{x}{4} \right)$
- (b) $\sqrt{2} \log \tan \left(\frac{\pi}{4} + \frac{x}{8} \right)$
- (c) $2 \sin \frac{x}{2} - 2 \cos \frac{x}{2}$
- (d) $2 \cos \frac{\pi}{2} - 2 \sin \frac{\pi}{2}$.

8. $\lim_{n \rightarrow \infty} \left[\tan 0 + \tan \frac{\pi}{4n} + \tan \frac{2\pi}{4n} + \dots + \tan \frac{n\pi}{4n} \right]$ is equal to

(a) $\frac{3}{\pi} \log 2$

(b) $\frac{\pi}{2} \log 3$

(c) $\frac{2}{\pi} \log 2$

(d) $\log 3$.

9. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$, then A^{-1} is

(a) $\begin{bmatrix} \frac{11}{3} & -3 & \frac{1}{3} \\ -\frac{7}{3} & 3 & -\frac{2}{3} \\ \frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix}$

(b) $\begin{bmatrix} 11 & -9 & 1 \\ -7 & 9 & -2 \\ -2 & -3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & -\frac{5}{2} & \frac{1}{2} \\ -2 & 4 & -1 \\ 1 & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$.

10. The values of δ and μ for which the following system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \delta z &= \mu \end{aligned}$$

have an infinite number of solutions are

(a) $\delta = 10$ and $\mu = 10$

(b) $\delta = 3$ and $\mu = 10$

(c) $\delta = 3$ and $\mu \neq 10$

(d) $\delta \neq 3$ and μ may have any value.

Part-II

20X4=80

11. A volcanic bomb of mass m is ejected vertically upward with an initial velocity of w_0 . Given negligible air resistance and g : acceleration due to gravity, the altitude (h) the volcanic bomb attains is

- (a) $h = w_0^2/2g$
- (b) $h = w_0^2/2gm$
- (c) $h = w_0/2g$
- (d) $h = w_0/g$

12. In making hydraulic models it is generally considered to be very important to observe Froude number scaling. The Froude number is given

as $F_r = \frac{U}{\sqrt{gd}}$ (U -velocity, d -density). What does Froude scaling imply

about the velocity, if the scale of the model is 1/100 that of the original.

- (a) 1/1
- (b) 1/10
- (c) 1/100
- (d) 1/1000.

13. Consider the stress matrix $[\sigma_{i,j}] = \begin{pmatrix} 4 & 2 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 8 \end{pmatrix}$.

The principal stresses ($\sigma_1, \sigma_2, \sigma_3$) are proportional to the determinant of the stress matrices, then principal stresses are given by:

- (a) 8, 4.385, -6.385
- (b) 8, 2.385, -6.385
- (c) 6, 4.385, -6.385
- (d) 8, 4.385, -1.385.

14. A spherical boulder (specific weight 27000 Nm^{-3} ; diameter 2.0 m) is observed to have half of its volume projecting out of a debris flow. The specific weight of the debris flow matrix is 20000 Nm^{-3} . What percentage of the total weight of the boulder is supported by buoyancy?

- (a) 27 percent
- (b) 47 percent
- (c) 37 percent
- (d) 0 percent.

15. Assume a plate moving at a velocity of 70 mm yr^{-1} , and an asthenosphere with a thickness of 200 km, viscosity μ of 10^{19} Pa s , a base having zero velocity and zero pressure gradient in the direction of movement. What is the shear stress at the base of the lithosphere?

- (a) 0.100 MPa
- (b) 0.099 MPa
- (c) 0.001 MPa
- (d) 0.111 MPa.

16. A coal seam is encountered in three bore holes (A, B & C) at depths of 100 m (A), 50 m (B) and 100 m (C) respectively. A, B and C are situated on the vertices of an equilateral triangle with sides of 100 m each. The true dip of the coal seam is

- (a) $\tan^{-1}(1)$
- (b) $\tan^{-1}(\sqrt{3})$
- (c) $\tan^{-1}(1/\sqrt{3})$
- (d) $\tan^{-1}(2/\sqrt{3})$.

17. Two quartz veins which were originally at right angles to each other are affected by a parallel sided ductile shear zone of width w . If one of the veins (I) was perpendicular to the shear zone wall before deformation, and the two veins make an angle of 30° with each other after shearing, the displacement of vein I across the shear zone is given by

- (a) $w \tan 30^\circ$
- (b) $\tan 30^\circ/w$
- (c) $w / \tan 60^\circ$
- (d) $w / \tan 30^\circ$.

18. Consider a tetragonal mineral prism with long axis of the prism as the z-axis, and diagonals of the prism cross-section as x-axis and y-axis. If the width of each prism face is given by 'a', the equation of the normal to the prism face is given by:

- (a) $x + y = \frac{a}{\sqrt{2}}$
- (b) $x - y = \frac{a}{\sqrt{2}}$
- (c) $x + y = a$
- (d) $x - y = 0$.

19. The equatorial radius of the Earth is 6378 km, while the polar radius is 6357 km. Assume shape of the Earth as an oblate spheroid. A ship sails along the equator between longitudes 70°E and 80°E . Another ship, starting from the same point, sails along 70°E meridian between latitudes 0° and 10°N . The difference in distance (km) travelled by the two ships is given by

- (a) $2\pi \cdot \frac{21}{360}$
- (b) $2\pi \cdot 21$
- (c) $2\pi \cdot \frac{21}{36}$
- (d) $2\pi \cdot \frac{21}{6}$.

20. A geologist makes a mistake in recording the latitude and longitude of the corners of a rectangular area bounded by parallels of latitude and longitude. He has recorded the corners of the area as P (S $15^\circ 00'$: W $78^\circ 00'$), Q (S $15^\circ 00'$: W $77^\circ 45'$), R (S $15^\circ 15'$: W $78^\circ 00'$), and S (S $15^\circ 15'$: W

78°05′). If PQ and PS are NOT diagonals of the rectangle, which of the following inference is correct with respect to error in recording?

- (a) Either P or Q is incorrect
- (b) Either Q or R is incorrect
- (c) Either Q or S is incorrect
- (d) Either R or S is incorrect.

21. Five hundred chert pebbles measured on a beach have mean sphericity of 0.71, standard deviation is 0.08. How many pebbles are expected to have sphericity values greater than 0.79, assuming the distribution is normal?

- (a) 92
- (b) 79
- (c) 40
- (d) 57.

22. Your colleague has reported the following percentages of clasts present in a sandstone sample: Quartz – 35.0%, K Feldspar – 27.5%, Plagioclase – 20.0%, Mica – 17.5%.

If your colleague claims that the values are accurate up to one decimal place then what is the minimum number of grains measured?

- (a) 1000
- (b) 924
- (c) 40
- (d) 100.

23. The axes of trough cross strata are oriented at 355°, 350°, 005°, 285°, 075°, 010°, 0°. What is the approximate mean paleocurrent direction?

- (a) North
- (b) South
- (c) North-west
- (d) North-east.

24. What is the slope of the land surface where the contours of 500m interval are separated by 1 cm in a 1:50000 scale toposheet?

- (a) 30°
- (b) 45°
- (c) 60°
- (d) 15° .

25. The length and width measurements of six well-preserved Mollusca shells are given in the following table:

Sp. No.	Width (cm)	Length (cm)
1	1	4
2	6	39
3	1.5	5.25
4	8.5	75.25
5	3	12
6	9	84

Which of the following statements is most appropriate for this data set?

- (a) There is a linear correlation between the length and width measurements.
- (b) The length and width measurements are negatively correlated.
- (c) There is no correlation between the length and width measurements.
- (d) There is a non-linear correlation between the length and width measurements.

26. The following table gives the relation between pairs of data values (X_i, Y_i) for $i = 1, \dots, 5$

$$X_i - 1, 2, 3, 4, 5$$
$$Y_i - 2, 4, 6, 8, 10$$

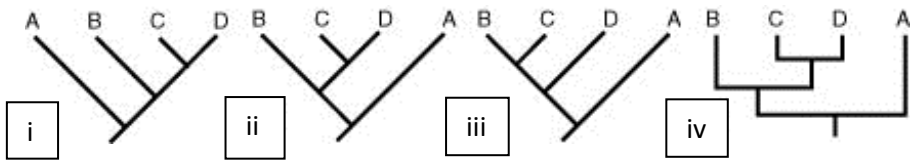
All the points lie on $Y = bX$ with slope b and correlation coefficient r . The values of b and r will be

- (a) $r = 2, b = 1$
- (b) $r = 1, b = 2$
- (c) $r = 2, b = 2$
- (d) $r = 1, b = 12$.

27. 200 g sample of Lawrencium was left in a container from 8.00 AM one morning until 2.00 PM next afternoon. If the remaining mass of Lawrencium was 25 g, what is the half-life of Lawrencium?

- (a) 30 hours
- (b) 10 hours
- (c) 5 hours
- (d) 1 day.

28. Of the cladograms shown below, which one shows a different evolutionary history from the others?



- (a) i
- (b) ii
- (c) iii
- (d) iv.

29. Consider the following reaction $2\text{H}^+ + 2\text{e}^- \rightleftharpoons \text{H}_2(\text{g})$. The Nernst equation for this reaction would be (given, $RT/F = 0.0257 \text{ V}$ at 25°C)

- (a) $E_h = 0.000 - 0.0592\text{pH}$
- (b) $E_h = 2.000 + 0.0592\text{pH}$
- (c) $E_h = 1.000 - 0.0592\text{pH}$
- (d) $E_h = 1.000 + 0.0592\text{pH}$

30. As per the data given below, approximately what percentage of localities has either *Nummulites* or *Discocyclus*?

Fossil present	Localities
<i>Nummulites</i>	241
<i>Discocyclus</i>	361
<i>Alveolina</i>	129
<i>Pallatospira</i>	45
<i>Globorotalia</i>	63
<i>Non</i>	84

- (a) 65
- (b) 26
- (c) 39
- (d) 75.